Free optimization

Find the extremes of this function and classify them

$$z = (x - y)^4 + (y - 1)^4 + 2$$

Solution

$$z'x = 4(x-y)^3 = 0$$
$$z'y = -4(x-y)^3 + 4(y-1)^3 = 0$$

From the first equation we get: $x = y$. From the second equation:

$$-4(y-y)^3 + 4(y-1)^3 = 0$$

Then:

$$4(y-1)^3 = 0$$
$$y = 1 = x$$

To verify if it is a minimum or maximum, we proceed with the second derivatives

$$z''_{xx} = 12(x-y)^2$$
$$z''_{yy} = 12(x-y)^2 + 12(y-1)^2$$
$$z''_{xy} = z''_{yx} = 0$$

Evaluating at the point:

$$z''_{xx} = 0$$
$$z''_{yy} = 12$$
$$z''_{xy} = z''_{yx} = 0$$

$$|H| = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

The Hessian is 0, but if we analyze the form of the expression, we can see that the terms raised to the fourth power:

$$z = \overbrace{(x-y)^4}^{\geq 0} + \overbrace{(y-1)^4}^{\geq 0} + 2$$

must be positive or equal to 0. Therefore, the lowest value the function can achieve is 2. We are thus dealing with a minimum.